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### I. INTRODUCTION

In order to characterize the dynamical properties of a net-\\ transformation of the state of t Transformation of the state of t i de la constant de la transformation of the state of t لين من العالمين والمعالمين والمعالمين والمعالمين والمعالمين والمعالمين والمعالمين والمعالمين والمعالمين والمعا معالم المعالمين والمعالمين والمعالمين والمعالمين والمعالمين والمعالمين والمعالمين والمعالمين والمعالمين والمعال definition of the state of the s to the state of the definition of the state of the s pair in the IT cortex on the basis of cross- and autocorrela-activities of an IT neuron pair and the spike train dynamics of the constituent neurons of the pair.

#### **II. EXPERIMENTAL PROCEDURE AND DATA ANALYSIS**

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$$\hat{R}_i = \hat{H}_i(t), \tag{1}$$

$$\hat{H}_i(t) = \langle S_i^k(t) \rangle_{tri}.$$
(2)

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$$\hat{R}_{ij}(\tau) = \langle [S_i^k(t) - \hat{H}_i(t)] [S_j^k(t+\tau) - \hat{H}_j(t+\tau)] \rangle_{tri}.$$
 (3)

The width of the time bin used in the calculation was 10 ms. The cross correlogram was estimated only for spike trains recorded from different electrodes.

$$\hat{R}_{12}^{sh}(\tau) = \langle \overline{[S_1^k(t) - \hat{H}_1(t)][S_2^{o(k)}(t+\tau) - \hat{H}_2(t+\tau)]} \rangle_{tri}, \quad (4)$$

where o(k) represents the trial order after the shuffle. This correlogram is called the shuffle correlogram. We shuffled the trial order 1000 times and thereby obtained a total of 1000 shuffle correlograms. On the basis of these shuffle correlograms, we estimated 95% confidence limits of the cross correlograms. If five consecutive bins in the cross correlograms exceeded the upper limit, then we regarded the peak as significant.

### **III. EXPERIMENTAL RESULTS**

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reliably extract spike activities of single neurons from the signals recorded at several sites. As a result, we obtained spike activities of 46 single neurons in total. From these 46 neurons, we obtained 57 neuron pairs. The number of neuron pairs is smaller than that of all possible combinations of the 46 neurons. This is because we did not record spike activities of all 46 neurons simultaneously.

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FIG. 2. Autocorrelograms of the neurons (a) and a comparison between the auto- and cross correlograms (b). In (a), solid line for autocorrelogram of one neuron; broken line for that of the other neuron. Each correlogram was normalized by the square of the average firing rate of each neuron. For display, the values at 0 time delay were set to 0. In (b), solid line for the cross correlogram shown in Fig. 1; broken line for the autocorrelogram represented by the solid line in (a). The cross correlogram was normalized by the product of the average firing rates of the neurons.

## IV. TWO-DIMENSIONAL POINT PROCESS MODULATED BY A TWO-STATE MARKOV PROCESS

#### A. Formulation

$$S_i(t) = s_i(t)X(t), \tag{5}$$

$$\langle s_i(t) \rangle = \nu_i, \tag{6}$$

$$[s_i(t) - \nu_i][s_i(t+\tau) - \nu_i]\rangle = f_i(\tau), \tag{7}$$

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tion, we assume that  $s_1(t)$  and  $s_2(t)$  are statistically independent, and X(t) and  $s_i(t)$  (i=1,2) are also statistically independent. The state space of X(t) is  $X(t) = \{0,1\}$ ; we call the states X(t) = 1 and X(t) = 0 the up and down states, respectively. The transition rate from the down to the up state is denoted by  $\lambda$ , while that from the up to the down state is denoted by  $\mu$ .

For this model, the average firing rate of each neuron  $R_i$  is given by

$$R_i = \nu_i \frac{\lambda}{\lambda + \mu}.$$
(8)

The auto-  $R_{ii}(\tau)$  (i=1,2) and the cross correlation  $R_{12}(\tau)$  functions are given by

$$R_{ii}(\tau) \approx \nu_i^2 \frac{\lambda \mu}{(\lambda + \mu)^2} e^{-(\lambda + \mu)|\tau|} (\tau_s < \tau), \qquad (9)$$

$$R_{12}(\tau) = \nu_1 \nu_2 \frac{\lambda \mu}{(\lambda + \mu)^2} e^{-(\lambda + \mu)|\tau|},$$
 (10)

where  $\tau_s$  indicates the time constant of the function  $f_i(\tau)$ . From Eqs. (9) and (10), we can see that after the normalization employed in Fig. 1, the auto- and cross correlations coincide, except at around the zero time delay. Thus, this model is consistent with the result that the auto- and cross correlograms of the pair coincide well except at around the zero time delay (Fig. 2).

### **B.** Parameter estimation

We estimated all the parameters of the model  $\{\nu_1, \nu_2, \lambda, \mu\}$ from the experimental data. For the neuron pair used in Figs. 1 and 2, average firing rates of the constituent neurons were estimated from the experimental data using Eq. (1). The estimated average firing rates were  $5.560 \times 10^{-3}$  and  $6.087 \times 10^{-3}$  ms<sup>-1</sup>. These rates correspond to Eq. (8). Thus, we obtained the following equations:

$$\nu_1 \frac{\lambda}{\lambda + \mu} = 5.560 \times 10^{-3} \text{ ms}^{-1},$$
 (11)

$$\nu_2 \frac{\lambda}{\lambda + \mu} = 6.087 \times 10^{-3} \text{ ms}^{-1}.$$
 (12)

Furthermore, by comparing the result of the least-squares fitting of the cross correlogram (Fig. 1) and Eq. (10), we obtained the equations

$$\lambda + \mu = 8.9 \times 10^{-3} \text{ ms}^{-1}, \tag{13}$$

$$\nu_1 \nu_2 \frac{\lambda \mu}{(\lambda + \mu)^2} = 2.6 \times 10^{-5} \text{ ms}^{-2}.$$
 (14)

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$$T_{up} = \frac{1}{\mu} = 2.6 \times 10^2 \text{ ms},$$
 (15)

$$T_{down} = \frac{1}{\lambda} = 2.0 \times 10^2 \text{ ms.}$$
 (16)

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#### C. Evaluation of the model

In order to examine whether our model well describes the experimentally observed spike trains of IT neuron pairs, we estimated the spike count distribution of two neurons from experimentally observed spike trains. We then statistically compared this distribution with that derived from our model using the parameters estimated from the experimental data.

For this purpose, we specified the point process  $s_i(t)$  as follows. The time constant of a narrow peak in the autocorrelogram is smaller than that of the two-state Markov process X(t). Thus, as long as we focus analysis on the two-state Markov process, the time constant of the narrow peak can be neglected, i.e., we can approximate the function  $f_i(\tau)$  with a Dirac  $\delta$  function. In addition, the inequality  $\nu_i \ll 1$  holds. On the basis of these considerations, we approximated the point process  $s_i(t)$  with a Poisson process.

Under this approximation, the spike count distribution of the two neurons during the interval [0,t] is given by the following equation:

$$P\{N_{1}(t) = n, N_{2}(t) = m\} = P_{nm}$$

$$= E\left[\frac{\left(\nu_{1}\int_{0}^{t}X(u)du\right)^{n}}{n!}\exp\left(-\nu_{1}\int_{0}^{t}X(u)du\right)$$

$$\times \frac{\left(\nu_{2}\int_{0}^{t}X(u)du\right)^{m}}{m!}\exp\left(-\nu_{2}\int_{0}^{t}X(u)du\right)\right],$$
(17)

where  $P\{\cdots\}$  represents the spike count distribution,  $N_i(t)$  the spike count of neuron *i* during the interval [0, t], and  $E[\cdots]$  the expectation value. The distribution given by Eq. (17) was calculated from 100 000 sets of two spike trains generated on the basis of the model using the parameters estimated from the experimental data. The calculated distribution was compared with that estimated from experimentally observed spike trains. In the comparison, we calculated a statistic  $\chi^2$  given by

$$\chi^{2} = \sum_{i=0}^{k} \sum_{j=0}^{l} \frac{(\hat{f}_{ij} - nP_{ij})^{2}}{nP_{ij}}.$$
 (18)

In this equation,  $f_{ij}$  denotes the experimentally observed frequency at spike counts *i* and *j*, *n* the number of trials, and *k* and *l* are the maximum spike counts of the two neurons, respectively.



FIG. 3. Probability distribution of  $\chi^2$ . The dashed line indicates  $\chi^2 = 10.3$ .

For a statistical test of  $\chi^2$ , we require its sample distribution. In order to estimate this distribution, we first generated sets of two spike trains on the basis of our model using the parameters estimated from the experimental data. The spike generation was repeated as many times (trials) as the experithe state of the s mental data and the spike count distribution that corresponds to the experimentally observed distribution was estimated from the generated spike trains. Subsequently, the parameters of the model were estimated from the generated spike trains in the same manner as in the preceding subsection. On the basis of the model using the estimated parameters, we gen- 0 to the state of t spike count distribution that corresponds to the model distribution. Finally, we calculated  $\chi^2$  from the calculated spike count distributions. We repeated this procedure 1000 times. As a result, we obtained 1000 samples of  $\chi^2$ . On the basis of these samples, we estimated the sample distribution of  $\chi^2$ (Fig. 3).

Figure 4 shows the experimentally observed spike count distribution of two neurons whose cross correlogram is shown in Fig. 1 and the distribution based on the model using the parameters estimated from the experimental data. As shown in Fig. 4, the two distributions are very similar. In fact,  $\chi^2$  was 10.3, providing  $P(\chi^2 > 10.3) = 0.27$  (Fig. 3). Thus, we cannot reject the null hypothesis that the experimentally observed spike count distribution is a sample distribution derived from the model. We obtained the same results for all the six pairs. These results suggest that spike trains of 35% of the IT neuron pairs that have significant correlation are well described by a two-dimensional Poisson process whose means are modulated by a two-state Markov process.

# V. TWO-DIMENSIONAL POISSON PROCESS MODULATED BY AN ORNSTEIN-UHLENBECK PROCESS

#### A. Formulation

Next, we examined whether spike trains of an IT neuron pair can be described by a two-dimensional Poisson process whose means are modulated by an OU process. When the change of firing rates of two neurons is described by an OU



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$$Y_i(t) = \sqrt{\alpha_i} \int_{-\infty}^t e^{-\kappa(t-s)} dB(s) + \beta_i, \qquad (19)$$

$$\langle dB(t)\rangle = 0, \tag{20}$$

$$\langle dB(t)dB(t')\rangle = \delta(t-t')dt dt', \qquad (21)$$

$$R_i = \beta_i. \tag{22}$$

Moreover, the auto- and cross correlations of the spike trains are given by

$$R_{ii}(\tau) = \frac{\alpha_i}{2\kappa} e^{-\kappa|\tau|} (\kappa \neq 0), \qquad (23)$$

$$R_{12}(\tau) = \frac{\sqrt{\alpha_1 \alpha_2}}{2\kappa} e^{-\kappa |\tau|}.$$
(24)

From these equations, we can see that this model can explain the result shown in Fig. 3 when the following equation holds:

$$\frac{\alpha_1}{\beta_1^2} = \frac{\alpha_2}{\beta_2^2}.$$
(25)

In addition, to explain the experimental data by this model, the fluctuation of the firing rate must be less than the average firing rate. This is because the firing rate cannot be : The set of the set of

$$\alpha_i \ll 2\kappa \beta_i^2, \tag{26}$$

because  $V_i$  is given by

$$V_i = \frac{\alpha_i}{2\kappa}.$$
 (27)

#### B. Parameter estimation and evaluation of the model

From the experimental data, we can estimate all the parameters of the model. The average firing rates of the neurons estimated from the experimental data were 5.560  $\times 10^{-3}$  and  $6.087 \times 10^{-3}$  ms<sup>-1</sup> (see Sec. IV B). Thus, from Eq. (22) we obtained the equations

$$\beta_1 = 5.560 \times 10^{-3} \text{ ms}^{-1},$$
 (28)

$$\beta_2 = 6.087 \times 10^{-3} \text{ ms}^{-1}.$$
 (29)

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$$\kappa = 8.9 \times 10^{-3} \text{ ms}^{-1}, \tag{30}$$

$$\frac{\sqrt{\alpha_1 \alpha_2}}{2\kappa} = 2.6 \times 10^{-5} \text{ ms}^{-2}.$$
 (31)

By solving Eqs. (28)–(31) and (25), we obtained  $\alpha_1$ =4.2 × 10<sup>-7</sup> ms<sup>-3</sup> and  $\alpha_2$ =4.6 × 10<sup>-7</sup> ms<sup>-3</sup>.

By using the estimated value of  $\alpha_i$  and Eqs. (28)–(30), we can estimate the right-hand side of Eq. (26) as follows:

$$2\kappa\beta_1^2 = 5.5 \times 10^{-7} \text{ ms}^{-3}, \qquad (32)$$

$$2\kappa\beta_2^2 = 6.6 \times 10^{-7} \text{ ms}^{-3}.$$
 (33)

From these equations, we can see that the inequality (26) does not hold. We obtained the same results for the six neuron pairs. Thus, this model does not describe spike trains of an IT neuron pair.

#### VI. DISCUSSION

In this study, we have quantitatively shown that there exist IT neuron pairs whose spike trains are well described by a two-dimensional Poisson process whose means are modulated by a common two-state Markov process. Raster plots of the neurons of a pair confirmed qualitatively that the modulations are not described by an OU process but by a two-state Markov process (Fig. 5). The plots show that in the spike train of each neuron there are two distinct periods: the period during which the neuron fires (firing period) and the period during which the neuron does not fire (nonfiring period). The firing and nonfiring periods are likely to correspond to the up and down states, respectively. The plots also confirmed that a two-state Markov process is common to the neurons of the



FIG. 5. Raster plots of the neurons of the pair whose cross correlogram is shown in Fig. 1. The vertical lines indicate spike timing. For each trial, spike timing of one neuron is indicated in the top row and that of the other neuron in the bottom row.

pair. The firing periods and the nonfiring periods of the neurons overlap well, respectively (Fig. 5).

What is the physiological meaning of the states? One of the interpretations is that the two activity states correspond to two states of membrane potential of a neuron [4,5]. For example, the membrane potential of a neuron in the striatum shows two states: the neuron fires during the up state (average potential  $-49.02 \pm 4.16$  mV), while it does not fire during the down state (average potential  $-71.51 \pm 3.81$  mV) [4]. The relation between the firing and the states of the membrane potential is very similar to our model. In the striatum, the average duration of the up state is  $422.57 \pm 84.81$  ms and that of the down state is  $313.01 \pm 43.20$  ms [4]. These values are comparable to the average durations of the up and down states of IT neurons we estimated. In addition, a correlated state transition of membrane potentials of neurons has been observed in the striatum and other brain areas [6-8]. This is also consistent with the correlated state transition in our model.

The sources of the two activity states and the underlying mechanism to generate a correlated state transition of IT neurons remain unknown. One possible source is the nonlinearity of a single neuron. The dynamics of the membrane potential of a single neuron is described by nonlinear equations. Thus, if an appropriate ion channel exists, the membrane potential can have two stable states [9]. Another possibility is that the two activity states of a neuron emerge only at the network level. For example, an ensemble of neurons in the network composed of excitatory neurons expressing the H current [10] and inhibitory neurons show a correlated transition between two states of membrane potential [11]. In this case, the correlated state transition between two neurons in a network can reflect the state transition of the network.

### VII. CONCLUSION

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broad peak in the cross correlogram of an IT neuron pair and revealed the relation between spike activities of the pair.

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